

Sample tasks from:

Geometry Assessments Through the Common Core (Grades 6-12)

A resource from
The Charles A Dana Center at
The University of Texas at Austin

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About the Dana Center Assessments

More than a decade ago, the Charles A. Dana Center began work on collections of assessment tasks that could be used by teachers at many grade levels to enable them to assess student learning continually as they enacted mathematics instruction. These assessments, developed in collaboration with mathematics educators, are designed to make clear to teachers, students, and parents what is being taught and learned about the most central mathematical concepts at each grade or in each course. Published by the Dana Center as a series of books (available for order here: <http://www.utdanacenter.org/products/math.php>), these collections of assessments eventually encompassed tasks for middle school, Algebra I, Geometry, and Algebra II.

These tasks have been used and tested by tens of thousands of educators and their students. Now the Dana Center has published a selection of these tasks in new editions: 118 tasks in *Algebra Assessments Through the Common Core (Grades 6-12)* and 57 tasks in *Geometry Assessments Through the Common Core (Grades 6-12)*.

The alignment to the Common Core State Standards for Mathematics of 164 existing Dana Center mathematics assessment tasks, and the development of 11 new tasks aligned to the standards, was made possible by a grant from Carnegie Corporation of New York and from the Bill and Melinda Gates Foundation. (Gates Foundation grant number OPP-48458 and Carnegie Corporation of New York grant number B 8312.) The statements made and views expressed are solely the responsibility of the authors.

We thank the Gates Foundation and Carnegie Corporation of New York for their support of Improving Mathematics and Literacy Instruction at Scale, a collaboration between the Aspen Institute (literacy) and the Dana Center (mathematics). We used funds from this grant to align mathematics assessments to the "Standards for Mathematical Content" and "Standards for Mathematical Practice" of the Common Core State Standards for Mathematics and to publish these assessments to this UDLN website. We thank these funders for their generous support of this work.

To support the adoption of the Common Core State Standards for Mathematics, the Dana Center is pleased to make available here two sample tasks from *Geometry Assessments Through the Common Core (Grades 6-12)*: "Whitebeard's Treasure" and "Walking the Archimedean Walk."

About Whitebeard's Treasure

Standard for Mathematical Practice #3 calls for students to construct viable arguments. Students should be able to use definitions in constructing arguments and justify their conclusions. In this task, students explore a context involving two quadrilaterals in a coordinate plane. Students construct the second quadrilateral from the first following specific directions, and are asked to describe the new quadrilateral. Students must use the definitions of special quadrilaterals to identify the type of quadrilateral they have created, and justify their answer using two different mathematical arguments.

About Walking the Archimedean Walk

Standard for Mathematical Practice #7 calls for students to look for and make use of structure. Specifically, students are expected to look closely for patterns and to make use of those patterns. In this task, students recreate Archimedes's approach to using patterns to investigate the value of π using a series of polygons inscribed in circles without using measurement. Students describe how patterns they see in the relationships between the diameter and the perimeter/circumference approach the value of π .

Whitebeard's Treasure

Whitebeard, the notorious pirate of the West Bay, buried treasure on Tiki Island over 200 years ago. Archeologists recently discovered a map showing the location of the treasure. The location has generated quite a bit of media attention, much to the dismay of the archeologists. In order to allow both the media and archeologists to work together, officials have decided to erect two fences around the location, allowing the media access to the site, yet allowing the archeologists room to work. One fence encloses the actual area where the archeologists will work. Another fence surrounds the enclosed dig area.

Descriptions of the fencing locations have been provided to the media so they may indicate accessible areas for their employees. Use the given information to draw and label a quadrilateral on graph paper indicating the location of the two fences.

1. Corners of the first fence are located at points A(11,3), B(3,-11), C(-13,-9) and D(-5,9). The media must stay within this fenced area. Connect the points in alphabetical order, and then join point D to Point A.
2. Find and label the midpoints of each segment of quadrilateral ABCD, showing all work. Label the midpoints of the segments as follows:

\overline{AB} has midpoint Q,

\overline{BC} has midpoint R,

\overline{CD} has midpoint S,

\overline{DA} has midpoint T.

3. Connect the four midpoints in alphabetical order to create a new quadrilateral QRST. This quadrilateral represents the fence surrounding the archeological dig site.
4. Quadrilateral ABCD was an ordinary quadrilateral, but QRST is a special one. Determine the special name for quadrilateral QRST, and justify your answer using coordinate geometry in two different ways.

Teacher Notes

CCSS Content Task

(6.G) **Solve real-world and mathematical problems involving area, surface area, and volume.**

3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

(G-CO) **Prove geometric theorems**

11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

(G-GPE) **Use coordinates to prove simple geometric theorems algebraically**

4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.*

6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

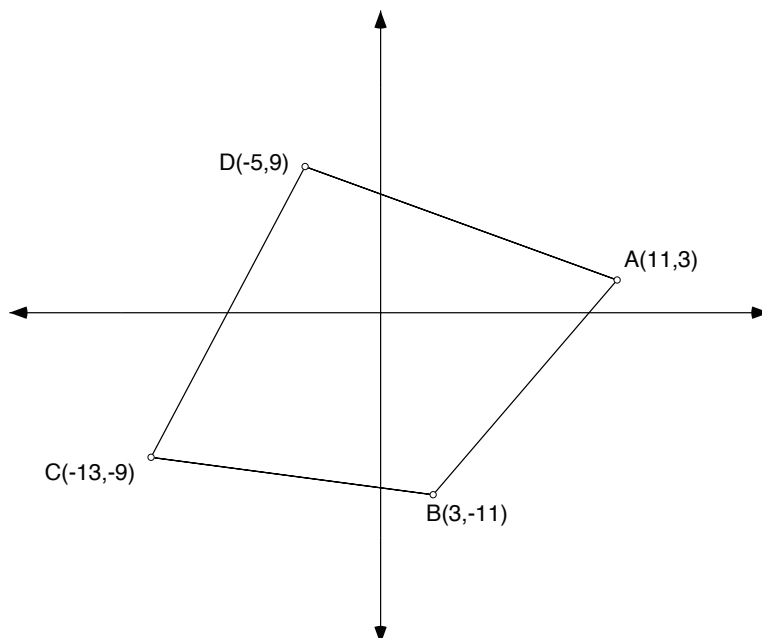
This performance task addresses the same mathematical concepts as the Wearable Art task later in this chapter. Whitebeard's Treasure gives the numerical coordinates. In Wearable Art the coordinates are given and the student must represent the situation using variable coordinates for the points. The teacher may choose to use one or both of these problems.

Scaffolding Questions:

- What is the formula for finding the midpoint of a line segment?
- Which of the quadrilaterals are special quadrilaterals?
- What are the characteristics of each special quadrilateral?
- What characteristics does quadrilateral QRST appear to possess that matches one of the special quadrilaterals?
- How can you prove these special characteristics?

Sample Solutions:

Quadrilateral ABCD is graphed as shown. This is the outer fence.



Chapter 1:

Coordinate Geometry

To find the midpoint of each segment of quadrilateral ABCD, use the midpoint formula.

The midpoint of the segment with endpoints (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$.

To find the midpoint of each segment, substitute the x and y values from the endpoints of the segment into the formula as follows:

Midpoint of \overline{AB} (Point Q)

$$\left(\frac{11 + 3}{2}, \frac{3 + (-11)}{2}\right) = (7, -4)$$

Midpoint of \overline{BC} (Point R)

$$\left(\frac{3 + (-13)}{2}, \frac{-11 + (-9)}{2}\right) = (-5, -10)$$

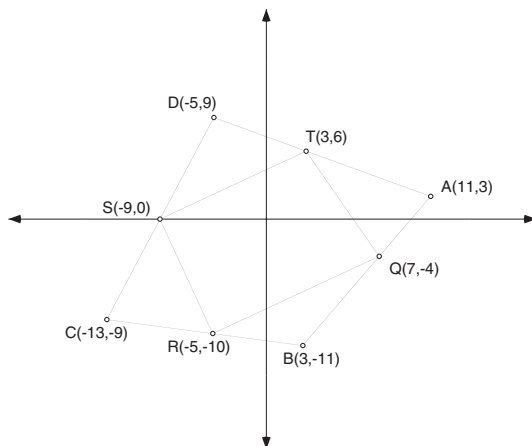
Midpoint of \overline{CD} (Point S)

$$\left(\frac{-13 + (-5)}{2}, \frac{-9 + 9}{2}\right) = (-9, 0)$$

Midpoint of \overline{DA} (Point T)

$$\left(\frac{-5 + 11}{2}, \frac{9 + 3}{2}\right) = (3, 6)$$

Graph the midpoints and connect them in alphabetical order to form a new quadrilateral QRST.



Standards for Mathematical Practice

3. Construct viable arguments and critique the reasoning of others.

Quadrilateral QRST would be the fence that encloses the archeologists' dig site.

Quadrilateral QRST appears to be a parallelogram because the opposite sides of the newly formed quadrilateral appear to be parallel. One way to prove that a quadrilateral is a parallelogram is to prove that both pairs of opposite sides are parallel. Lines that have the same slope are parallel lines.

Use the slope formula:

The slope of the line through points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

The slope of \overleftrightarrow{RS} is $\frac{0 - (-10)}{-9 - (-5)} = \frac{10}{-4} = -\frac{5}{2}$

The slope of \overleftrightarrow{QT} is $\frac{6 - (-4)}{3 - 7} = \frac{10}{-4} = -\frac{5}{2}$

$\overleftrightarrow{RS} \parallel \overleftrightarrow{QT}$ because both lines have the same slope.

The slope of \overleftrightarrow{ST} is $\frac{6 - 0}{3 - (-9)} = \frac{6}{12} = \frac{1}{2}$

The slope of \overleftrightarrow{RQ} is $\frac{-4 - (-10)}{7 - (-5)} = \frac{6}{12} = \frac{1}{2}$

$\overleftrightarrow{ST} \parallel \overleftrightarrow{RQ}$ because both lines have the same slope.

Quadrilateral QRST is a parallelogram by definition because both pairs of opposite sides are parallel.

Another way to show that QRST is a parallelogram is to prove that both sides of opposite sides are congruent (using the distance formula to find the lengths of each side).

Chapter 1:
Coordinate Geometry

$$QR = \sqrt{(7 - (-5))^2 + (-4 - (-10))^2} = \sqrt{144 + 36} = \sqrt{180} = 6\sqrt{5}$$

$$ST = \sqrt{(-9 - 3)^2 + (0 - 6)^2} = \sqrt{144 + 36} = \sqrt{180} = 6\sqrt{5}$$

$$QR = ST = 6\sqrt{5}$$

$$RS = \sqrt{(-5 - (-9))^2 + (-10 - 0)^2} = \sqrt{16 + 100} = \sqrt{116} = 2\sqrt{29}$$

$$QT = \sqrt{(7 - 3)^2 + (-4 - 6)^2} = \sqrt{16 + 100} = \sqrt{116} = 2\sqrt{29}$$

$$RS = QT = 2\sqrt{29}$$

Both pairs of opposite sides of quadrilateral QRST are congruent. Therefore, it is a parallelogram.

Extension Questions:

- Use algebra to find the point of intersection of the diagonals of quadrilateral QRST.

To find the point where the diagonals intersect, the equations of lines \overleftrightarrow{RT} and \overleftrightarrow{SQ} must be identified and then used to find the point of intersection.

The slope of \overleftrightarrow{RT} is $\frac{6 - (-10)}{3 - (-5)} = \frac{16}{8} = 2$

The slope of \overleftrightarrow{SQ} is $\frac{0 - (-4)}{-9 - 7} = \frac{4}{-16} = -\frac{1}{4}$

The equation of \overleftrightarrow{RT} is $(y - 6) = 2(x - 3)$ or $y = 2x$

The equation of \overleftrightarrow{SQ} is

$$y = -\frac{1}{4}(x - (-9))$$

$$y = -\frac{1}{4}x - \frac{9}{4}$$

The point where the diagonals intersect can be found by using linear combination.

$$y = 2x$$

$$y = -\frac{1}{4}x - \frac{9}{4}$$

$$2x = -\frac{1}{4}x - \frac{9}{4}$$

$$8x = -1x - 9$$

$$9x = -9$$

$$x = -1$$

$$y = 2x = 2(-1) = -2$$

The point of intersection is $(-1, -2)$.

- Use coordinate geometry to prove the diagonals of quadrilateral QRST bisect each other.

The midpoint of \overline{QS} is $\left(\frac{7+(-9)}{2}, \frac{-4+0}{2}\right) = (-1, -2)$

The midpoint of \overline{RT} is $\left(\frac{-5+3}{2}, \frac{-10+6}{2}\right) = (-1, -2)$

The midpoints of the segment are the same point as the intersection point. The diagonals bisect each other.

Chapter 1:
Coordinate Geometry

Student Work Sample

Field Test Teacher's Comment:

This was the first performance task I had the students do using a poster. I enjoyed the poster and I feel most of the students did, too. I would like to have done this problem during the quadrilaterals section and will do so next year. One thing I did different this time was to have the students write on the back of their solution guide exactly what to put on their posters for the three criteria we emphasized.

The teacher emphasized three criteria from the Geometry Solution Guide found in the introduction to this book. On the back of one student's solution guide were these notes:

Shows an understanding of the relationships among elements

- Statement showing how the elements are related.
- Can the history teacher understand your steps?

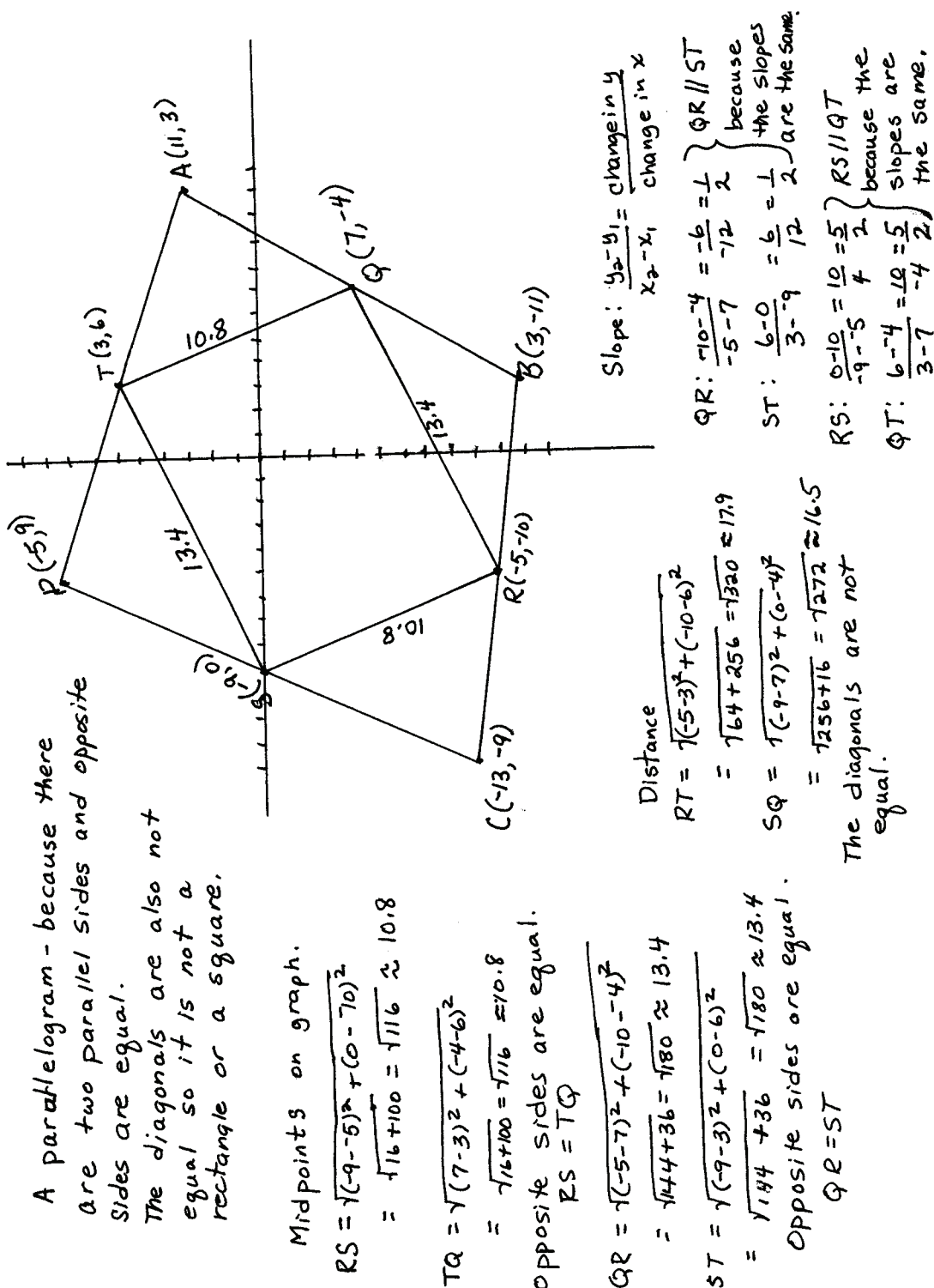
Makes an appropriate and accurate representation of the problem using correctly labeled diagrams

- Drawing the pictures
- Make appropriate markings on the picture

Communicates clear, detailed, and organized solution strategy

- Step by step details that can be followed
- Don't plug in a number without showing why/how
- Must have justification
- Explaining your thinking!!!

A copy of the poster from this student's group appears on the next page.



Walking the Archimedean Walk

Most geometry students know where the value of π comes from—their calculators. Most geometry students probably also realize that the number their calculator gives them is really an approximation of the value of π —the constant ratio between a circle’s circumference and its diameter:

$$\pi = \frac{C}{d}$$

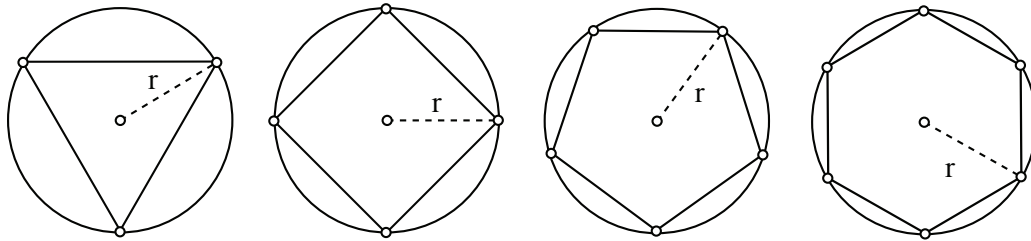
During the course of human history, diverse cultures throughout the world were aware of this constant ratio. The attempt to fix its exact value has been a vexing problem that has occupied many mathematical minds over the centuries. (See, for example, *A History of Pi* by Petr Beckmann, St. Martin’s Griffin, 1976.)

In our Western cultural tradition, the historical record tells us that Archimedes was the first person to provide a mathematically rigorous method for determining the value of π .

In this assessment, you will have to retrace his footsteps in order to demonstrate a solid understanding of where that number comes from when you push the “ π ” button on your calculator.

A logical starting place for determining π is to measure the circumferences and diameters of many circles and then calculate the ratio C/d based upon those measurements. You may have done a measurement activity like this in your geometry class. Archimedes realized that this method would always be limited by the precision of the people doing the measuring and by the accuracy of the measuring devices they were using. He sought a way to fix the value of π that was based upon direct calculation rather than upon measurement.

Archimedes's approach involved inscribing regular polygons in circles. He then considered what " π " would be for each of the inscribed regular polygons. You will model his approach by examining the figures below in the problems for this assessment.



Problem 1

1. Use construction tools, circular geoboards, circular dot paper, or appropriate geometry technology to inscribe an equilateral triangle in a circle with a radius of 2 units.
2. Make appropriate calculations (no measuring allowed!) in order to determine the value of π based upon an inscribed equilateral triangle.

Problem 2

1. Use construction tools, circular geoboards, circular dot paper, or appropriate geometry technology to inscribe a square in a circle with a radius of 2 units.
2. Make appropriate calculations (no measuring allowed!) in order to determine the value of π based upon an inscribed square.

Problem 3

1. Use construction tools, circular geoboards, circular dot paper, or appropriate geometry technology to inscribe a regular hexagon in a circle with a radius of 2 units.
2. Make appropriate calculations (no measuring allowed!) in order to determine the value of π based upon an inscribed regular hexagon.

Problem 4

1. Complete the table below, and summarize your findings.

Number of sides of the inscribed polygon	Measure of the central angle	Perimeter of inscribed polygon	Diameter of inscribed polygon	Approximation for π
3			4	
4			4	
6			4	

2. Does this method overestimate or underestimate the value of π ? Will this method result in an exact value for π ?
3. Write a few sentences and provide a diagram to answer question 2.

Problem 5

Write a few sentences explaining the basics of the method Archimedes used.



Notes

CCSS Content Task

(G-CO) **Make geometric constructions**

13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

(G-C) **Understand and apply theorems about circles**

3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Standards for Mathematical Practice

5. Use appropriate tools strategically.

7. Look for and make use of structure.

Chapter 2:

Patterns, Conjecture, and Proof

Teacher Notes

The next performance task, Talk the Archimedean Talk, is an extension of this task. It requires the student to repeat this activity for a dodecagon.

Scaffolding Questions:

Problems 1, 2, and 3

- What special right triangle is formed by the radius and the apothem of the inscribed polygon?

Problem 4

- Will the perimeter of the inscribed polygon be greater than, less than, or equal to the circumference of the circle?
- Will the ratio $\frac{\text{perimeter}}{\text{diameter}}$ be greater than, less than, or equal to the ratio $\frac{\text{circumference}}{\text{diameter}}$?
- Will the perimeter of the inscribed polygon ever be equal to the circumference of the circle?

Problem 5

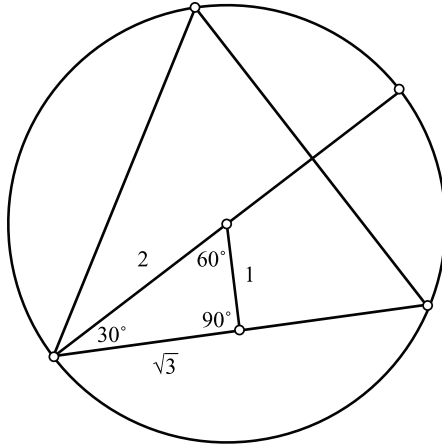
- If you consider the inscribed regular polygon to be a “primitive circle,” what measure in the polygon corresponds to the circumference of the circle?
- If you consider the inscribed regular polygon to be a “primitive circle,” what measure in the polygon corresponds to the diameter of the circle?
- What will happen to the ratio $\frac{\text{perimeter}}{\text{diameter}}$ as the number of sides of the polygon increases?

Chapter 2:
Patterns, Conjecture, and Proof

Sample Solutions:

Problem 1

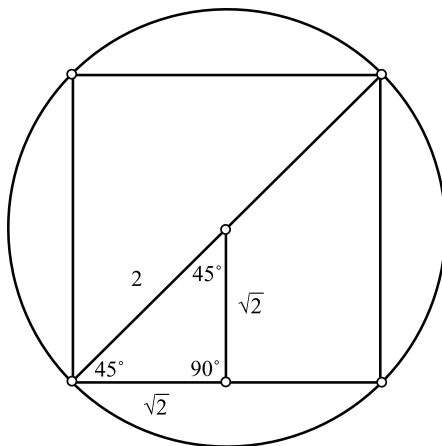
1.



2. $\frac{\text{perimeter}}{\text{diameter}} = \frac{6\sqrt{3}}{4} = \frac{3\sqrt{3}}{2} \approx 2.5981$

Problem 2

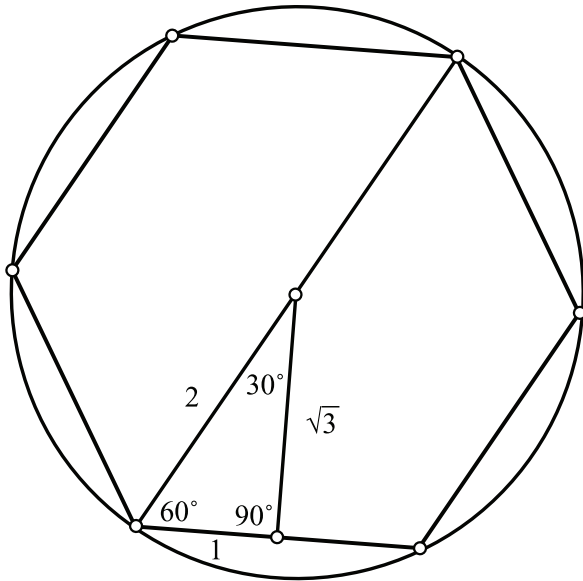
1.



2. $\frac{\text{perimeter}}{\text{diameter}} = \frac{8\sqrt{2}}{4} = 2\sqrt{2} \approx 2.8284$

Problem 3

1.



2. $\frac{\text{perimeter}}{\text{diameter}} = \frac{12}{4} = 3$

Problem 4

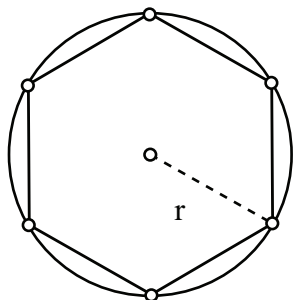
1.

Number of sides of the inscribed polygon	Measure of the central angle	Perimeter of inscribed polygon	Diameter of inscribed polygon	Approximation for π
3	120°	$6\sqrt{3}$	4	2.5981
4	90°	$8\sqrt{2}$	4	2.8284
6	60°	12	4	3.000

2. This method underestimates the value of π . The perimeter of the inscribed regular polygon is always less than the circumference of the circle it is inscribed within.

Chapter 2:
Patterns, Conjecture, and Proof

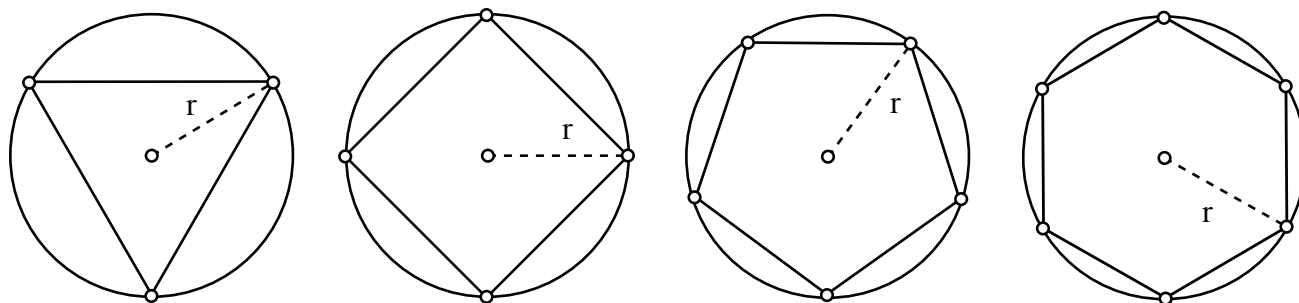
3.



The perimeter of the polygon is always less than the circumference of the circle, but the diameters for both polygon and circle are equal. This means that the ratio $\frac{\text{perimeter}}{\text{diameter}}$ is less than the ratio $\frac{\text{circumference}}{\text{diameter}}$.

As the number of sides of the polygon increases, the ratio $\frac{\text{perimeter}}{\text{diameter}}$ will get closer and closer to the value of π . It will never result in the exact value for π because no matter how many sides the inscribed polygon has, its perimeter will always be less than the circumference of the circle with the same center and radius.

Problem 5



Each inscribed regular polygon can be considered an approximation of the circle it is inscribed within. Consider the diameter of a polygon to be twice the radius of the inscribed circle. Therefore, the ratio of the polygon's perimeter to its "diameter" can be considered an approximation of the ratio of the circle's circumference to its diameter, π .

As the number of sides in the regular polygon increases, the ratio $\frac{\text{perimeter}}{\text{diameter}}$ gets closer and closer to the value of π . The method yields successively more accurate approximations of π .

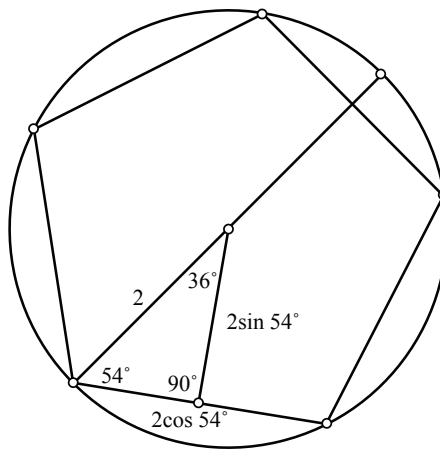
Extension Questions:

- Based on the chart in Problem 4, between what two values should an estimate of π , based on a pentagon with radius 2, fall?

The value should be between the values for a quadrilateral and a hexagon, or between 2.8284 and 3.000.

- Use construction tools or geometry software to construct a regular pentagon inscribed in a circle of radius 2. Then calculate an estimate for π based on the inscribed pentagon.

Geometry software was used to construct the figure.



Use trigonometry to solve this problem.

$$\frac{\text{perimeter}}{\text{diameter}} = \frac{20 \cos 54}{4} = 5 \cos 54 \approx 2.9398$$

The approximation for π is 2.9398.